College of the Holy Cross, Fall 2008
Math 131 - Practice Exam 3

The problems here are only a sample of possible exam questions. To fully prepare for the test, make sure you understand the examples discussed in class, as well as the homework problems.

I. Find y' and simplify.

(a)
$$y = \ln(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)$$

(b)
$$y = (e^{2x} + 2)^3$$

(c)
$$y = \frac{x+1}{3x^4 - 1}$$

(d)
$$y = \frac{\sin(x)}{1 + \cos(x)}$$

(e)
$$y = \tan^{-1}(e^{5x})$$

(f)
$$xy^2 - 3y^3 + 2x^4 = 2$$

(g)
$$y = \cos(x)^{x^3}$$

(h)
$$y = \frac{\pi^2 + \tan(e^\pi) - 2x^e}{2}$$

II. The quantity of a reagent present in a chemical reaction is given by $Q(t) = t^3 - 3t^2 + t + 30$ grams at time t seconds for all $t \geq 0$.

(a) Over which intervals with $t \geq 0$ is the amount increasing? decreasing?

(b) Over which intervals is the rate of change of Q increasing? decreasing?

III. A spherical balloon is being inflated at 20 cubic inches per minute. When the radius is 6 inches, at what rate is the radius of the balloon increasing? At what rate is the surface area increasing? (The volume of a sphere of radius r is $V = \frac{4\pi r^3}{3}$ and the surface area is $4\pi r^2$.)

IV. All parts of this question refer to $f(x) = 4x^3 - x^4$.

(a) Find and classify all the critical numbers of f using the First Derivative Test.

(b) Over which intervals is the graph $y = f(x)$ concave up? concave down?

(c) Sketch the graph $y = f(x)$.

(d) Find the absolute maximum and minimum of $f(x)$ on the interval $[1, 4]$.

V. All parts of this question refer to the function $f(x) = x^{2/3} - \frac{1}{5}x^{5/3}$.

(a) Find all the critical numbers of $f(x)$.

(b) Find all the inflection points of $f(x)$.

(c) For which of the critical numbers here is the Second Derivative Test applicable? Why? Determine the type of each such critical number using the Second Derivative Test.

VI. Water is leaking out of a conical tank at a rate of 1 m^3/min. The tank has height 6m and the radius of the top is 2m. Initially the tank is full. How fast is the water level dropping when the height of the water in the tank is 2m? (The volume of a cone of height h and base circle of radius r is $\frac{1}{3}\pi r^2 h$.)

VII. a) Find the linear approximation of the function $f(x) = \ln(x)$ at $x = 1$.
b) Using the linear approximation in part a), estimate $\ln(1.1)$.
c) Is your estimate larger or smaller than the actual value? Justify your answer.

VIII. Find the equation of the tangent line to the curve $x^2 + 4xy + y^2 = 13$ at the point $(2, 1)$.