1. Suppose you have to construct a dog kennel as shown below. The kennel is a rectangle with four pens of equal size. The outside of the kennel and the three internal partitions are made of fencing that costs $2 per foot. You have a budget of $200. Find the dimensions $x$ and $y$ that will maximize the total area enclosed.

Solution: The fencing costs $2 per foot and you have $200, so you can buy 100 feet of fence. There are 5 sections of length $x$ and 2 of length $y$, so the constraint is

$$100 = 5x + 2y.$$  

The area is $xy$. Solving the constraint equation for $y$ we get

$$A = xy = x\left(\frac{100 - 5x}{2}\right) = 50x - \frac{5}{2}x^2.$$  

Take the derivative and set it equal to zero to find the dimension $x$ that minimizes the area:

$$A'(x) = 50 - 5x = 0 \Rightarrow x = 10.$$  

Solving for $y$ gives $y = 25$, so the dimensions are $x = 10, y = 25$.

2. A home-improvement store sells 2000 sheets of plywood per year at a steady rate. In order to keep enough plywood in inventory, the store will place several orders of the same size at equally spaced intervals throughout the year. The ordering cost for each order is $200, and the carrying cost, based on the average number of sheets of plywood in inventory, is $5 per year for each sheet of plywood. Determine the number of orders that should be placed per year in order to minimize the total inventory cost.

Solution: Let $r$ be the number of orders per year and let $x$ be the number of sheets of plywood per order, so

$$rx = 2000 \Rightarrow x = \frac{2000}{r}$$

The ordering cost is

$$200r$$

and the carrying cost is

$$5 \cdot \frac{x}{2}$$

since on average there are $x/2$ sheets of plywood in inventory. The total inventory cost is therefore

$$C = 200r + 5 \cdot \frac{x}{2} = 200r + \frac{5}{2} \cdot \frac{2000}{r} = 200r + 5000r^{-1}$$

Take the derivative and set it equal to zero to find the $r$ that minimizes the total cost:

$$C' = 200 - 5000r^{-2} = 0 \Rightarrow 200 = \frac{5000}{r^2} \Rightarrow r^2 = 25$$

so the number of orders that should be placed to minimize the total cost is $r = 5$. 


3. Use the product rule to find $y'$, if

$$y = (x^2 + 3x - 2)(\sqrt{x} + \frac{1}{x})$$

You do not need to simplify your answer.

*Solution:* Let $f(x) = (x^2 + 3x - 2)$, $g(x) = (\sqrt{x} + \frac{1}{x})$, so

$$y' = f'g + g'f = (2x + 3)(\sqrt{x} + \frac{1}{x}) + (x^2 + 3x - 2)\left(\frac{1}{2}x^{-1/2} - \frac{1}{x^2}\right)$$

4. Use the quotient rule to find the slope of the tangent line to

$$y = \frac{x^2 + 1}{x^2 - 1}$$

at the point $(2, 5/3)$.

*Solution:* Let $f(x) = x^2 + 1, g(x) = x^2 - 1$, so

$$y' = \frac{f'g - g'f}{g^2} = \frac{2x(x^2 - 1) - 2x(x^2 + 1)}{(x^2 - 1)^2} = \frac{-4x}{(x^2 - 1)^2}$$

Plug in $x = 2$ to get

$$y' = -\frac{8}{9}$$

so the slope of the tangent line at $(2, 5/3)$ is $m = -8/9$. 