Mathematics 132 – Calculus for Physical and Life Sciences 2
Exam 3 – Review Sheet
April 2008

General Information

As announced in the course syllabus, the third midterm exam of the semester will be given at 7:00pm on Wednesday, April 23 (note different time) in O'Neil 112. The format will be similar to that of the first midterm and the exams from last semester.

- **Bring a photo ID to the exam**.
- The exam will be designed to take an hour but you will have an extra 30 minutes to work and check your solutions.
- You will be given a TI-30 scientific calculator for the exam which does NOT have graphing capabilities so be prepared to answer questions without your personal calculator. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of cell phones, iPods, and all other electronic devices *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the last exam (Problem Sets 7, 8, and 9), namely the following material from sections 7.1 - 7.5, and 8.1 - 8.7 of Stewart:

1. Differential equations (Chapter 7)
   (a) solutions of a differential equation (know how to determine whether a given function is or is not a solution of a given equation; for this type of question it will not be necessary to derive a general solution),
   (b) direction fields (be prepared to sketch a simple direction field by hand, and/or identify computer-generated plots),

(c) Euler's method for approximating solutions,

(d) separable equations and solution by separation of variables and integration,

(e) exponential growth and decay problems; other growth and decay problems (be prepared to set up a differential equation matching a verbal description, solve it, and use the solution to answer questions),

(f) logistic equations – know the form

$$y' = ky\left(1 - \frac{y}{M}\right)$$

of the logistic equation, the properties of the solutions, and the analytic formula

$$y = \frac{M}{1 + be^{-kt}}$$

for the solutions.

2. Sequences and Series (Chapter 8)

   (a) the concepts of sequences and series (and the difference between them!), the concepts of convergence for sequences and for series (and the difference between them!)

   (b) key classes of examples such as geometric series, telescoping series, the harmonic series, p-series, etc.

   (c) general convergence tests: the Integral Test, the Alternating Series Test, the Test for Divergence, the Ratio Test for absolute convergence (in addition to using these on specific examples, be prepared to give careful *statements* of the theorems concerning each test – what hypotheses are necessary, what conditions imply convergence or divergence, and the conclusion in each case; also be prepared for questions like 8 from Problem Set 8, where you must give a reason why a particular test does not apply for a particular example).

   (d) power series, radius of convergence, interval of convergence

   (e) power series representing functions, Taylor series and Taylor polynomials.

Important Note: Some of the problems on this exam (in particular solving separable differential equations and applying the Integral Test for series) will require you to set up and compute integrals to find the quantity that is asked for. In addition to knowing how to set up the required integral, *any* of the methods of integration tested on the first exam (i.e. basic rules, u-substitution, integration by parts, trigonometric substitution, partial fractions, or consultation of a table of integrals) might be required to evaluate the integral. Again, there is a high degree of cumulativeness built into this material.

There will be a review for the exam in class on Tuesday, April 22.

Review Problems

- Section 7.1/1,3,5,9,11
- Section 7.2/1,3-6,9,23
- Section 7.3/1,3,5,7,9,11,33,35
- Section 7.4/3,9,11,13,17
- Section 7.5/1,3,7
- Section 8.1/3,5,9,11,13
- Section 8.2/9,11,13,15,19,21,25
- Section 8.3/7,21 (use the integral test)
- Section 8.4/19, 21, 23, 25
- Section 8.5/5,7,9
- Section 8.6/3,5,7,9,11,21
- Section 8.7/1,3 (Maclaurin series = Taylor series with $a = 0$),11,13,15,41,43

Sample Exam Questions

This list is much longer than the actual exam will be (to give you some idea of the range of different questions that might be asked).

I. (A) Show that for any constant c, $y = x^2 + \frac{c}{x^2}$ is a solution of the differential equation

$$y' = 4x - \frac{2}{x}y.$$

(B) All parts of this question refer to the differential equation

$$y' = y(4 - y)$$

(1) Sketch the slope field of this equation, showing the slopes at points on the lines $y = 0, 1, 2, 3, 4, 5$

(2) On your slope field, sketch the graph of the solution of the equation with $y(0) = 1$.

(3) Use Euler's method to approximate the solution of this equation with $y(0) = 1$ for $0 \leq x \leq 1$ using $n = 4$.

(4) This is a separable equation, find the general solution and determine the constant of integration from the initial condition $y(0) = 1$.

(C) Find the general solutions of the following differential equations

(1) $y' = \dfrac{y}{x(x+1)}$

(2) $y' = \dfrac{\sqrt{1-x^2}}{e^{2y}}$.

(D) Newton's Law of Cooling states that the rate at which the temperature of an object changes is proportional to the difference between the object's temperature and the surrounding temperature. A hot cup of tea with temperature 100°C is placed on a counter in a room maintained at constant temperature 20°C. Ten minutes later the tea has cooled to 76°C. How long will it take to cool off to 45°C? (Express Newton's Law as a differential equation, solve it for the temperature function, then use that to answer the question.)

II. (A) Does the sequence $a_n = n \ln(1+n)$ converge? Why or why not? Does the infinite series $\sum_{n=1}^{\infty} n \ln(1+n)$ converge? Why or why not?

(B) Use the Integral Test to determine whether or not

$$\sum_{k=1}^{\infty} \frac{k}{e^k}$$

converges.

(C) Use the Ratio Test to determine whether or not

$$\sum_{k=0}^{\infty} \frac{3^n}{n!}$$

converges.

(D) Determine (with justification!) whether or not the following series converge:

$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}, \qquad \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{\pi^{2n}}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^{1.01}}.$$

(E) Let $f(x) = \sqrt{1+x} = (1+x)^{1/2}$. Find the 4th degree Taylor polynomial of f centered at $a = 0$. Find a factorial expression for the general term of the Taylor series.

(F) Consider the geometric series $f(x) = \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$.

(1) Use series manipulations to find the Taylor series of $xf'(x)$.

(2) Use series manipulations to find the Taylor series of $-\ln(1-x)$.

(3) Find the radius of convergence of the series in part (b), and investigate convergence at the endpoints.

(4) Use parts (1) and (2) to *evaluate* the sums of the series $\sum_{k=1}^{\infty} \frac{1}{k \cdot 2^k}$ and $\sum_{k=1}^{\infty} \frac{k}{2^k}$.

(G) For each of the given power series, find the interval of convergence.

$$f(x) = \sum_{n=1}^{\infty} \frac{(2x)^n}{\sqrt{n}}, \qquad g(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{n \cdot 3^n}.$$

(In particular, give the radius of convergence, and investigate convergence at the endpoints.)

(H) The second degree Taylor polynomial of $f(x)$ at $a = 0$ is $p_2(x) = c + bx + ax^2$. What can you say about the signs of a, b, c if you know $f(x)$ is increasing and concave down at $x = 0$?