Math 132: Calculus for Physical and Life Sciences 2
Problem Set 10
Due Tuesday, April 29, 2008, at the beginning of class.

General Directions: You must show all work for credit on these problems.

1. The tread life (in thousands of miles) of a certain make of tire is a continuous random variable x with probability density function

$$f(x) = 0.02e^{-0.02x}$$

for $x \geq 0$.

   (a) Find the probability that a randomly selected tire of this make will have a tread life between $40,000$ and $60,000$ miles.

   (b) Find the probability that a selected tire will have a tread life of at most $30,000$ miles.

   (c) Find the probability that a selected tire will have a tread life of at least $70,000$ miles.

   (d) Find the mean tread life of this brand of tires.

2. Suppose t is the time (in hours) it takes for calculus students to complete their final exam. Assume that all students finish within 3 hours and that the probability density function for the time t is

$$f(t) = \begin{cases} \frac{4x^3}{81} & \text{if } 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Verify that $f(t)$ is a probability density function.

   (b) What proportion of the students take between 1.5 and 2.5 hours to complete the exam?

   (c) What is the mean time for students to complete the exam?

   (d) What is the median time for students to complete the exam?

   (e) Find the variance and standard deviation of the random variable t associated with $f(t)$.

3. The distribution of scores on IQ exams is often modeled by a normal distribution with mean $\mu = 100$ and standard deviation $\sigma = 15$.

   (a) Give the formula for the normal probability density function that fits this description.

   (b) Estimate the fraction of the population with IQ scores between 115 and 120 by applying the Midpoint Riemann sum approximation for the appropriate integral. Use $n = 5$ subintervals in the Riemann sum.

(c) Estimate the fraction of the population with IQ scores between 140 and 150 by the same method as in part (b).

4. Let μ and $\sigma^2 > 0$ be any two real constants.

   (a) Show that the normal probability density function

   $$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

   has exactly one critical number at $x = \mu$, and that $f(x)$ has a local maximum at $x = \mu$.

   (b) Show that $f(x)$ has inflection points at $x = \mu + \sigma$ and $x = \mu - \sigma$.

   (c) Give qualitative sketches of $y = f(x)$ with $\mu = 4$ and $\sigma^2 = 4$, and the cumulative distribution function for this normal distribution.