College of the Holy Cross, Fall Semester, 2021
Math 302 (Professor Hwang), Meeting 06
Local Geometric Theorems

Exercise 06.1: (Proposition 2.3) Suppose α is a space path. Prove α has curvature identically zero if and only if α traces a line. Hints: One direction is easy. For the other, assume the curvature is zero and conclude the valocity is constant.

Exercise 06.2: (Proposition 2.4) Suppose α is a space path.

(a) Prove α has torsion identically zero if and only if α lies in a plane. Hints: If α is a plane path, show its binormal is constant, and use this fact to calculate the torsion. Conversely, use the Frenet equations to show that if $\tau = 0$, the binormal is constant, say $\mathbf{B}_0$. Then show the function $\alpha(s) \cdot \mathbf{B}_0$ is constant; conclude α lies in a plane.

(b) Prove that if in addition α has constant curvature $\kappa > 0$, then α is part of a circle of radius $a = 1/\kappa$. Hint: Show the vector-valued function $\alpha(s) + \frac{1}{\kappa}\mathbf{N}(s)$ is constant.

Exercise 06.3: Exercise 1.2.2, page 18.

Exercise 06.4: Exercise 1.2.4, page 18.

Exercise 06.5: Exercise 1.2.8, page 18.