

College of the Holy Cross, Fall Semester, 2021
Math 302, Section 01 (Professor Hwang)
Course Information Sheet

Contact Information

Office: 339 Swords Hall

Office Hours: W 11-11:50, F 10:00-10:50, or by appointment

email: `ahwang -at- holycross -dot- edu`

web: <http://mathcs.holycross.edu/~ahwang/teach/302/index.html>

About the Course

Differential geometry is the use of calculus to study shape. Our objects of study are smooth curves and surfaces, subsets of space on which we can do calculus. There are two notions of “curvature”. *Extrinsic* curvature measures the way an object bends in space, but cannot be detected by measurement within the object itself. *Intrinsic* curvature measures the failure of small triangles to have total interior angle π , and makes sense independently of an object’s situation in space.

A smooth curve has only extrinsic curvature; no local measurement “inside” a curve can distinguish a circle from a straight line. A cylinder or a cone are extrinsically curved, but intrinsically flat. A sphere is intrinsically curved, a fact with consequences for cartography.

The primary technical tools are linear algebra (dot products, cross products, linear operators, and eigenvalues), and multivariable differentiation and integration, particularly the chain rule and the fundamental theorems.

Grading

My goals for you in the course are that you enjoyably use calculus to study the shape of curves and surfaces, and explain your work clearly in writing. The course grade has three components: a proof portfolio, two in-class tests, and the final exam.

Proof Portfolio Each Monday I’ll assign computations and proof questions for write-up. These should be turned in for commenting by the following Monday. These problem sets are not numerically scored, but instead allow you to work with the material and receive feedback. Some questions will be marked with a topic category. As the semester progresses, you’ll submit final drafts of ten questions of your choosing, one from each topic category. These write-ups will count for 50% of your course grade. Please see the proof portfolio page <http://mathcs.holycross.edu/~ahwang/teach/302/prob.html> for information, expectations, and grading.

Midterms There are two in-class midterm tests, scheduled for Friday, October 1 and Friday, November 12. Each midterm is worth 15% of your course grade. If you have a conflict due to an athletic event, illness, or a family emergency, notify me and your Class Dean immediately.

Final Exam The final exam is worth 20% of the course grade. The College has not yet scheduled the final exam.

Do not make travel plans that conflict with the midterms or final exam!

Meeting Schedule

The following is the syllabus for the term. Please see the course web pages for a more detailed list of topics. Any substantial variations from this schedule will be announced by email or in class.

W	Sep 1	Section 1.1	Plane paths, velocity, acceleration
F	Sep 3	Section 1.1	Space paths, arc length
M	Sep 6	Section 1.1	Differential equations
W	Sep 8	Section 1.2	The Frenet frame
F	Sep 10	Section 1.2	The Frenet equations
M	Sep 13	Section 1.2	Local geometric theorems
W	Sep 15	Section 1.3	The fundamental theorem of curves
F	Sep 17	Section 1.3	Curves on the unit sphere
M	Sep 20	Section 1.3	Spherical triangles, geography
W	Sep 22	Section 2.1	Surfaces
F	Sep 24	Section 2.1	The unit normal field, regularity
M	Sep 27	Section 2.1	The first fundamental form
W	Sep 29	Section 2.2	The Gauss map and shape operator
F	Oct 1		Midterm 1
M	Oct 4	Section 2.2	Examples and theorems, normal sections
W	Oct 6	Section 2.2	Principal and asymptotic directions
F	Oct 8	Section 2.2	Curvature, Meusnier's formula
M	Oct 18	Section 2.2	Surfaces of rotation
W	Oct 20	Section 2.2	Surfaces of rotation
F	Oct 22	Section 2.3	Coordinate frames, Christoffel symbols
M	Oct 25	Section 2.3	The shape operator
W	Oct 27	Section 2.3	The Codazzi and Gauss equations
F	Oct 29	Section 2.3	The Theorema Egregium, Clairaut surfaces
M	Nov 1	Section 2.3	Theorems about Gaussian curvature
W	Nov 3	Section 2.3	Liebmann's theorem
F	Nov 5	Section 2.4	Parallelism, paper surface geometry
M	Nov 8	Section 2.4	Covariant differentiation, parallel transport
W	Nov 10	Section 2.4	Geodesics
F	Nov 12		Midterm 2
M	Nov 15	Section 3.1	Moving frames, Clairaut surfaces
W	Nov 17	Section 3.1	Geodesic curvature, holonomy
F	Nov 19	Section 3.1	The local Gauss-Bonnet theorem
M	Nov 22	Section 3.1	The Gauss-Bonnet theorem
M	Nov 29	Section 3.1	Polyhedra
W	Dec 1	Section 3.2	Hyperbolic geometry
F	Dec 3	Section 3.3	Differential forms and exterior calculus
M	Dec 6	Section 3.3	The Cartan structure equations
W	Dec 8	Section 3.3	Gaussian curvature revisited
F	Dec 10		Review