College of the Holy Cross, Spring Semester, 2017
Math 242 (Professor Hwang)
Quiz 3 March 20, 2017

1. In each part, assume $f(x) = 2x - 5$.

   (a) If $x_0 = 3$ and $\varepsilon = 0.1$, find the largest $\delta > 0$ for which $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

   (b) If $x_0 = 3$ and $\varepsilon > 0$ is arbitrary, find the largest $\delta > 0$ for which $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

   (c) If x_0 and $\varepsilon > 0$ are arbitrary, find the largest $\delta > 0$ for which $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

2. Assume $f(x) = mx + b$. If x_0 and $\varepsilon > 0$ are arbitrary, find a $\delta > 0$ for which $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

3. In each part, assume $f(x) = x^2$.

   (a) If $x_0 = 3$ and $\varepsilon = 0.1$, find the largest $\delta > 0$ for which $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

   (b) If $x_0 = 3$ and $0 < \varepsilon < 1$ is arbitrary, find the largest $\delta > 0$ for which $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

   (c) If $x_0 \neq 0$ and $0 < \varepsilon < x_0^2$ are arbitrary, find the largest $\delta > 0$ for which $|x - x_0| < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

4. In each part, assume $f(x) = \sqrt{x}$ for $x \geq 0$.

   (a) If $x_0 = 0$ and $\varepsilon = 0.01$, find the largest $\delta > 0$ for which $0 \leq x - x_0 < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

   (b) If $x_0 = 0$ and $\varepsilon > 0$ are arbitrary, find the largest $\delta > 0$ for which $0 \leq x - x_0 < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

   (c) If $x_0 > 0$ and $0 < \varepsilon < |x_0|$ are arbitrary, find the largest $\delta > 0$ for which $0 \leq x - x_0 < \delta$ implies $|f(x) - f(x_0)| < \varepsilon$.

Answers:
1. (Each part) $\delta = \frac{1}{2}\varepsilon$.
2. If $m = 0$, every $\delta > 0$ works; if $m \neq 0$, then $\delta = \varepsilon/m$ (or anything smaller) works.
3. (b) We have $|x^2 - 9| < \varepsilon$ if and only if $9 - \varepsilon < x^2 < 9 + \varepsilon$, which holds if $\sqrt{9 - \varepsilon} < x < \sqrt{9 + \varepsilon}$. The largest positive δ is

$$\delta = \min(\sqrt{9+\varepsilon} - 3, 3 - \sqrt{9 - \varepsilon}) = \sqrt{9 + \varepsilon} - 3.$$

(c) Similarly to (b), $\delta = \sqrt{x_0^2 + \varepsilon} - |x_0|$.
4. (b) $\delta = \varepsilon^2$.
(c) $\delta = |(\sqrt{x_0} - \varepsilon)^2 - x_0| = x_0 - (\sqrt{x_0} - \varepsilon)^2$.