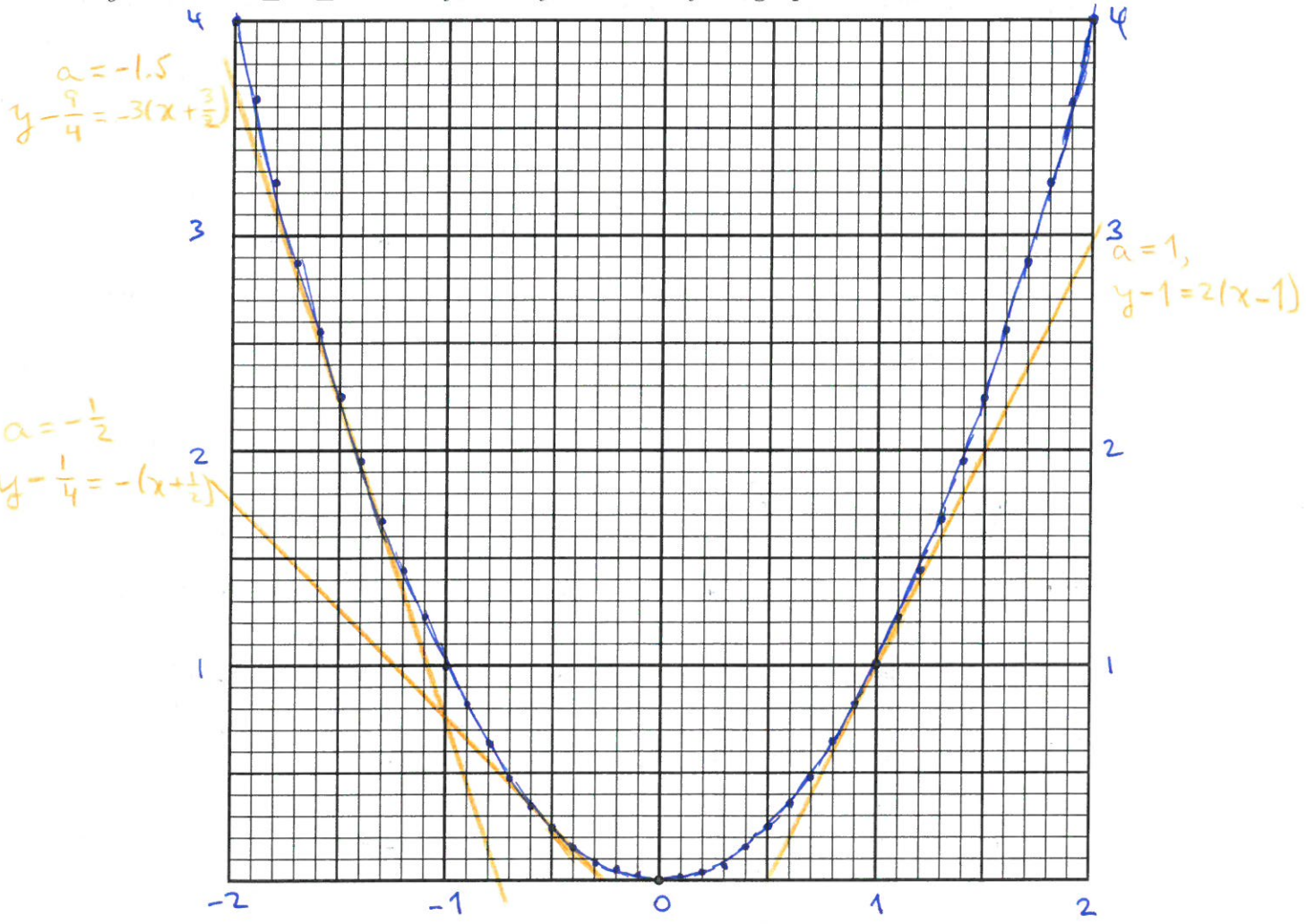
College of the Holy Cross
Math 135 (Calculus I)
Worksheet 3: Quadratic Polynomials

1. Values for x and $y = x^2$ are given.

x	y	x	y	x	y
0.0	0.00	0.7	0.49	1.4	1.96
0.1	0.01	0.8	0.64	1.5	2.25
0.2	0.04	0.9	0.81	1.6	2.56
0.3	0.09	1.0	1.00	1.7	2.89
0.4	0.16	1.1	1.21	1.8	3.24
0.5	0.25	1.2	1.44	1.9	3.61
0.6	0.36	1.3	1.69	2.0	4.00

Carefully plot the resulting 21 points in the grid provided. (The origin is marked at bottom. Assume the darkest squares are one unit on a side.) Connect the dots, obtaining the graph $y = x^2$ for $0 \le x \le 2$. Use symmetry to extend your graph to $-2 \le x \le 0$.



2. Pick a real number a between -2 and 2, and carefully draw the line of slope $m = 2a$ through the point (a, a^2) in the graph above. Repeat for a few different values of a. What do you notice?

Each line is tangent to the parabola at (a, a^2)

3. Suppose $a \neq 0$, b, and c are real numbers, and that x is a real number satisfying

$$ax^2 + bx + c = 0. \qquad (*)$$

Use the given steps to solve for x: (i) Multiply both sides of (*) by $4a$; (ii) Add $b^2 - 4ac$ to each side; (iii) Factor the left-hand side as a perfect square; (iv) Solve for x.

You should have found $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Memorize this formula.

(i) $4a^2x^2 + 4abx + 4ac = 0$

(ii) $4a^2x^2 + 4abx + b^2 = b^2 - 4ac$

(iii) $(2ax + b)^2 = b^2 - 4ac$

(iv) $2ax + b = \pm\sqrt{b^2 - 4ac}$

$2ax = -b \pm \sqrt{b^2 - 4ac}$, $\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

4. Factor the quadratics, and solve.

(a) $x^2 - 3x = 0$.
(b) $y^2 - 10 = 0$.
(c) $u^2 - 3u - 10 = 0$.
(d) $t^2 - 6t + 9 = 0$.
(e) $z^2 + 6z = -5$.
(f) $4r^2 - 8r + 3 = 0$.

(a) $x(x-3) = 0$, $x = 0, 3$

(b) $(y - \sqrt{10})(y + \sqrt{10}) = 0$, $y = \pm\sqrt{10}$

(c) $(u-5)(u+2) = 0$, $u = -2, 5$

(d) $(t-3)^2 = 0$, $t = 3$

(e) $(z+1)(z+5) = 0$, $z = -1, -5$

(f) $(2r-3)(2r-1) = 0$

$r = \frac{3}{2}$ or $r = \frac{1}{2}$.

5. Solve by factoring if possible, using the quadratic formula otherwise.

(a) $x^2 - x = 1$.
(b) $x^2 - x = 2$.
(c) $3z^2 - 2z - 5 = 0$.
(d) $s^6 - 4s^3 = 2$.
(e) $t^2 - 2\sqrt{2}t + 2 = 0$.
(f) $(u^2 - 4)^2 - 3(u^2 - 4) + 2 = 0$.

Answers: (a) $x = \frac{1 \pm \sqrt{5}}{2}$

(b) $x = -1, 2$

(c) $z = -1, \frac{5}{3}$

(d) $s = \sqrt[3]{2 \pm \sqrt{2}}$

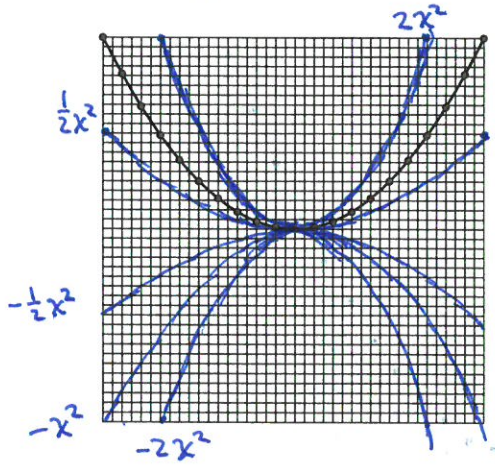
(e) $t = \sqrt{2}$

(f) $(u^2 - 4) = 1$ or 2, $\therefore u^2 = 5$ or 6

$u = \pm\sqrt{5}$ or $\pm\sqrt{6}$.

6. The graph $y = x^2$, $-1 \leq x \leq 1$, is shown at left. In the same grid, sketch the graphs

$y = 2x^2$; $y = \frac{1}{2}x^2$; $y = -x^2$; $y = -2x^2$; $y = -\frac{1}{2}x^2$.



7. The graph $y = x^2$, $-1 \leq x \leq 1$, is shown at right. In the same grid, sketch the graphs

$y = x^2 - 1$; $y = x^2 - \frac{1}{2}$; $y = 1 - x^2$;