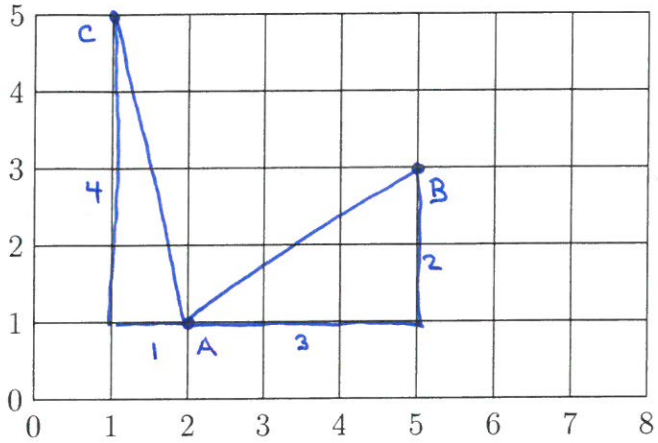
College of the Holy Cross
Math 135 (Calculus I)
Worksheet 2: Distance, Lines

1. (a) In the grid, plot the points $A = (2, 1)$ and $B = (5, 3)$. Draw the right triangle whose sides are parallel to the coordinate axes, whose hypotenuse is the segment $\overline{AB}$, and which lies below this segment. Find the lengths of the legs of this triangle.



Leg lengths:
Horizontal 3
Vertical 2

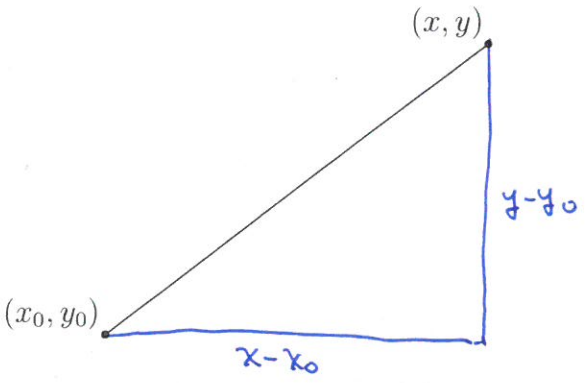
(b) Recall that if a right triangle has legs a and b and hypotenuse c, then $a^2 + b^2 = c^2$. Use this to calculate the hypotenuse of the triangle you sketched in part (b). Express your answer as a square root. (Do not evaluate numerically.) Hypotenuse $= \sqrt{3^2+2^2} = \sqrt{13}$

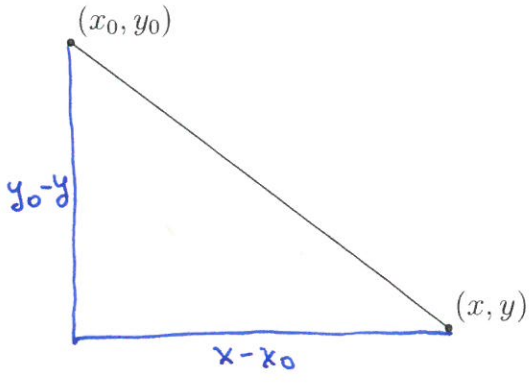
(c) In the grid, plot the point $C = (1, 5)$. Use the same reasoning as in (a) and (b) to find the distance between A and C. Without using a calculator: Which point is farther from A: B or C?

Horizontal 1
Vertical 4
Hypotenuse $= \sqrt{1^2+4^2} = \sqrt{17} > \sqrt{13}$: C is farther from A than B.

2. Suppose $A = (x_0, y_0)$ and $B = (x, y)$ are points in the Cartesian plane. Just as in the preceding problem, we may construct a right triangle with hypotenuse $\overline{AB}$.

(a) In each diagram below, sketch such a right triangle, and express the lengths of its legs in terms of A and B.

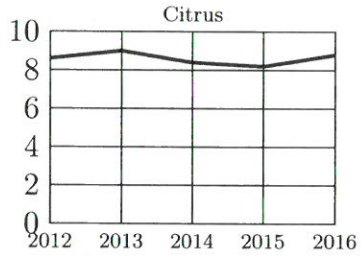


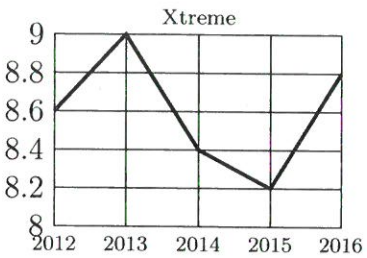


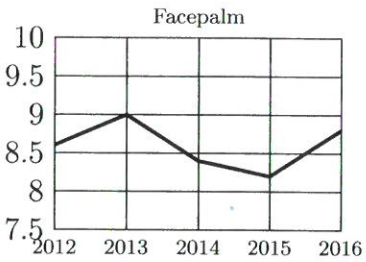
(b) Find a formula for the length of the hypotenuse of the triangles in part (a). (You should get the same formula.) This is the *distance formula* in the Cartesian plane.

$\sqrt{(x-x_0)^2 + (y-y_0)^2}$

3. Each graph below shows the annual profit of an imaginary company, in millions of dollars, between 2012 and 2016. Answer the questions based on the graphs.







(a) Which company has the largest maximum profit over 2012–2016?

Not a trick question, but a cautionary tale: All three data sets are identical.

(b) Which company is most consistently profitable over 2012–2016?

(c) Which company has the greatest volatility over 2012–2016?

4. On a piece of graph paper, sketch the lines with the following equations. Suggestion: Use a different color for each part.

(a) $y = 2x$,
$y = 2x + 3$,
$y = 2x - 1$.

(b) $y = -2x$,
$y = -2x + 3$,
$y = -2x - 1$.

(c) $y = \frac{1}{2}x$,
$y = \frac{1}{2}x + 3$,
$y = \frac{1}{2}x - 1$.

(d) $y = -\frac{1}{2}x$,
$y = -\frac{1}{2}x + 3$,
$y = -\frac{1}{2}x - 1$.

See graph paper

5. Find a point-slope equation $y - y_0 = m(x - x_0)$ for the line satisfying the given conditions:

(a) Slope 3 passing through the point $(-4, 5)$. $y - 5 = 3(x+4)$ or $y = 3x + 17$

(b) Slope 0 passing through the point $(-4, 5)$. $y - 5 = 0$ or $y = 5$

(c) Passing through $(-4, 5)$ and $(2, 1)$. Hint: First find the slope.

$\text{slope} = \frac{1-5}{2-(-4)} = \frac{-4}{6} = -\frac{2}{3}$

$y - 1 = -\frac{2}{3}(x-2)$ or $y = -\frac{2}{3}x + \frac{7}{3}$

6. Give the slope-intercept equation $y = mx + b$ of each line in the preceding question.