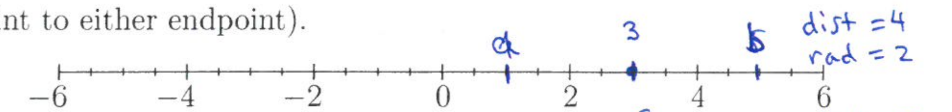
College of the Holy Cross
Math 135 (Calculus I)
Worksheet 1: Numbers and Coordinates

1. The term *percent* means "out of one hundred". A horizontal bar over a digit or group of digits means that pattern repeats forever. Fill in the missing entries in the table (the third row is done for you), and plot each number on the number line as accurately as you can.

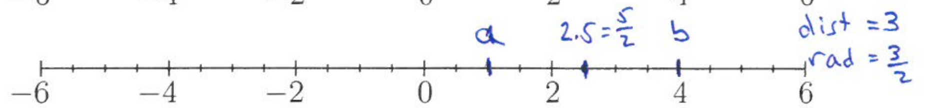
Decimal	Percent	Fraction	
0.125	12.5%	$\frac{1}{8}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0.2	20%	$\frac{1}{5}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0.25	25%	$\frac{1}{4}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
$0.\overline{33}$	$33\frac{1}{3}\%$	$\frac{1}{3}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0.375	37.5%	$\frac{3}{8}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0.5	50%	$\frac{1}{2}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0.6	60%	$\frac{3}{5}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0.625	62.5%	$\frac{5}{8}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
$0.6\overline{6}$	$66\frac{2}{3}\%$	$\frac{2}{3}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
0.75	75%	$\frac{3}{4}$	0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

2. For each pair of numbers a and b, (i) Plot them as points on the number line; (ii) Find and plot their midpoint; (iii) Find the distance between them and the "radius" of the interval (the distance from the midpoint to either endpoint).

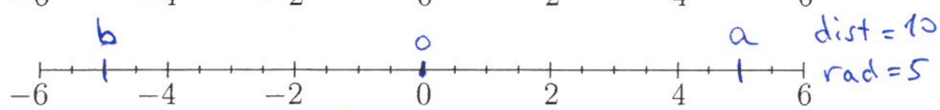
(a) $a = 1$, $b = 5$.



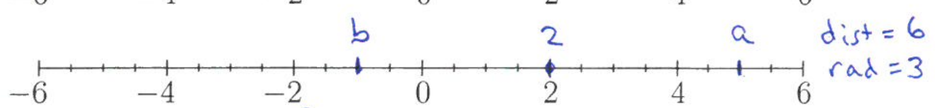
(b) $a = 1$, $b = 4$.



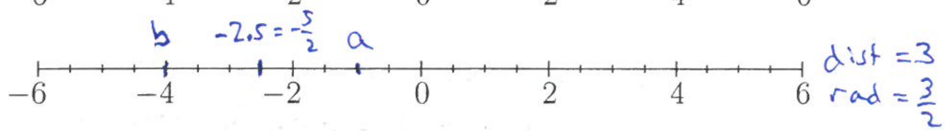
(c) $a = 5$, $b = -5$.



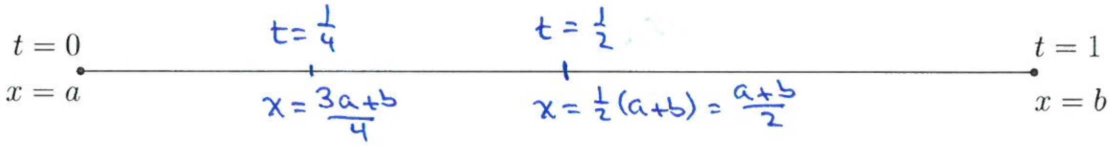
(d) $a = 5$, $b = -1$.



(e) $a = -1$, $b = -4$.



3. Let a and b be real numbers with $a < b$. Suppose a point particle traveling at constant speed starts at $x = a$ when $t = 0$, and arrives at $x = b$ when $t = 1$.



(a) Sketch the position when $t = \frac{1}{2}$, and find x in terms of a and b (and t).

(b) Sketch the position when $t = \frac{1}{4}$, and find x in terms of a and b (and t).

(c) For general t, find the position x in terms of a, b and t.
Hint: The displacement $x - a$ from the left endpoint is proportional to t; that is, $x - a = mt$ for some constant m. For which t do you know x? What does this tell you about m?

At $t = 1$, $x = b$, so $b - a = m \cdot 1 = m$. Substituting, $x - a = (b-a)t$,

or $\boxed{x = a + (b-a)t}$

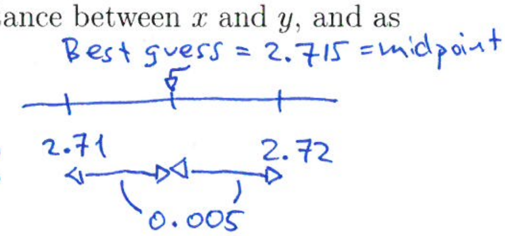
(d) Show that $x = (1-t)a + tb$, a *weighted average* of a and b.

From (c), $x = a + bt - at$

$= (1-t)a + bt.$

4. Suppose we play the following game: I think of a real number y between 2.71 and 2.72. You pick a real number x. Your score for the round is the distance between x and y, and as in golf, low scores are better.

(a) What should be your guess? Explain.



The midpoint $\frac{1}{2}(2.71 + 2.72) = 2.715$
is "closest to every point of the interval."

(b) For the guess in part (a), how large might your score be?

If $2.71 \le y \le 2.72$, then $|y - 2.715| \le 0.005$,
see diagram above.

(c) If we play this game several times and I choose my numbers randomly, what would your expected average score be?

The expected average score is (i.e. turns out to be) the average of 0 (best possible) and 0.005 (worst possible), i.e. $\boxed{0.0025.}$

(d) Suppose $a < b$ are numbers. Generalize parts (a)–(c) if I pick y between a and b.

The best guess is the midpoint, $\frac{1}{2}(a+b) = \frac{a+b}{2}$.

This is accurate to an error at most $\frac{1}{2}|b-a|$.

The expected score is half the maximum score, or $\frac{1}{4}|b-a|$.