

**College of the Holy Cross**  
**Math 135** (Calculus I)  
**Worksheet 11: Patching Functions**

1. Suppose

$$f(x) = \begin{cases} 4 - x^2 & x < 1, \\ a + \frac{b}{x} & 1 \leq x. \end{cases}$$

- (a) Find conditions on  $a$  and  $b$  so that  $f$  is continuous at 1.
- (b) Find conditions on  $a$  and  $b$  so that  $f$  is differentiable at 1.

2. Suppose

$$f(x) = \begin{cases} 4 - x^2 & x < 2, \\ a + \frac{b}{x} & 2 \leq x. \end{cases}$$

- (a) Find a condition on  $a$  and  $b$  so that  $f$  is continuous at 2.
- (b) Find a condition on  $a$  and  $b$  so that  $f$  is differentiable at 2.

3. Suppose

$$f(x) = \begin{cases} a(x+2) & x \leq -2, \\ 4 - x^2 & -2 < x < 1, \\ b + \frac{c}{x^2} & 1 \leq x. \end{cases}$$

- (a) Find conditions on  $a$ ,  $b$  and  $c$  so that  $f$  is continuous.
- (b) Find conditions on  $a$ ,  $b$  and  $c$  so that  $f$  is differentiable.

**Answers**  
1. Continuous if  $a + b = 3$ ; differentiable if  $a = 1$  and  $b = 2$ .  
2. Continuous if  $a + \frac{2}{b} = 0$ ; differentiable if  $a = -8$  and  $b = 16$ .  
3. Continuous if  $b + c = 3$  ( $a = \text{anything}$ ); differentiable if  $a = 4$ ,  $b = 2$ ,  $c = 1$ .