College of the Holy Cross
Math 135 (Calculus I)
Worksheet 11: Patching Functions

1. Suppose
$$f(x) = \begin{cases} 4 - x^2 & x < 1, \\ a + \frac{b}{x} & 1 \le x. \end{cases}$$

   (a) Find conditions on a and b so that f is continuous at 1.

   (b) Find conditions on a and b so that f is differentiable at 1.

2. Suppose
$$f(x) = \begin{cases} 4 - x^2 & x < 2, \\ a + \frac{b}{x} & 2 \le x. \end{cases}$$

   (a) Find a condition on a and b so that f is continuous at 2.

   (b) Find a condition on a and b so that f is differentiable at 2.

3. Suppose
$$f(x) = \begin{cases} a(x+2) & x \le -2, \\ 4 - x^2 & -2 < x < 1, \\ b + \frac{c}{x^2} & 1 \le x. \end{cases}$$

   (a) Find conditions on a, b and c so that f is continuous.

   (b) Find conditions on a, b and c so that f is differentiable.