**College of the Holy Cross**
**Math 135** (Calculus I)
**Worksheet 9: Logarithms**

In the equation $y = 10^x$, we usually think of $y$ as a function of $x$. We could, however, view the exponent $x$ as a function of $y$. We call $x$ the *base* 10 *logarithm* or *common logarithm* of $y$, and write $x = \log_{10} y$.

**1.** (a) Find $\log_{10} 100$ and $\log_{10} 1,000,000$. Hint: If $10^x = 100$, then $x = ?$

(b) Find $\log_{10} \sqrt{10}$ and $\log_{10} \sqrt[3]{10}$. Hint: If $10^x = \sqrt{10}$, then $x = ?$

(c) Find $\log_{10} \frac{1}{10}$, $\log_{10} \sqrt{1000}$.

(d) Suppose $\log_{10} y = 4.38$. Evaluate $\log_{10}(100y)$, $\log_{10} \frac{1}{y}$, and $\log_{10} y^2$. About how large is $y$? Particularly, how many digits does $y$ have?

**2.** (a) Recall that $y_1 y_2 = 10^{x_1} \times 10^{x_2} = 10^{x_1 + x_2}$. By taking logarithms, show

$$\log_{10}(y_1 y_2) = \log_{10}(y_1) + \log_{10}(y_2).$$

(b) Recall that $y^p = (10^x)^p = 10^{px}$. By taking logarithms, show that

$$\log_{10} y^p = p \log_{10} y.$$

(c) Recall that $2^{10} = 1024 \approx 1000$. Explain why $\log_{10} 2$ is a bit larger than 0.3.

**3.** Use the log table to find the indicated logarithms.

(a) $\log_{10} 2$ and $\log_{10} 5$. What do you notice about their sum. Can you explain?

(b) $\log_{10} 20$. Hint: $20 = 2 \times 10$. $\log_{10} 200$. $\log_{10} 2000$. $\log_{10} 2,000,000$.

(c) $\log_{10} \dfrac{1}{5}$. Hint: $\dfrac{1}{5} = \dfrac{2}{10} = 2 \times 10^{-1}$. $\log_{10} \dfrac{1}{50}$. $\log_{10} \dfrac{1}{5,000,000}$.

(d) $\log_{10} 2$. $\log_{10} 4$. $\log_{10} 8$. $\log_{10} 16$. How are these related? Why?

(e) $\log_{10} 2.718$. Hint: Linear interpolation.

**4.** For each positive real number $b \neq 1$, there is a *base b logarithm*, satisfying $x = \log_b y$ precisely when $y = b^x$. The logarithm with base $b = e \approx 2.71828$ is called the *natural logarithm*: $x = \ln y$ precisely when $y = e^x$.

**5.** (a) Find the power law $f(x) = cx^r$ satisfying $f(2) = 7$ and $f(5) = 28$.
Hint: Use the natural logarithm to solve for $r$, then express $c$ in terms of $r$.

(b) Find the power law satisfying $f(2) = 28$ and $f(5) = 7$.

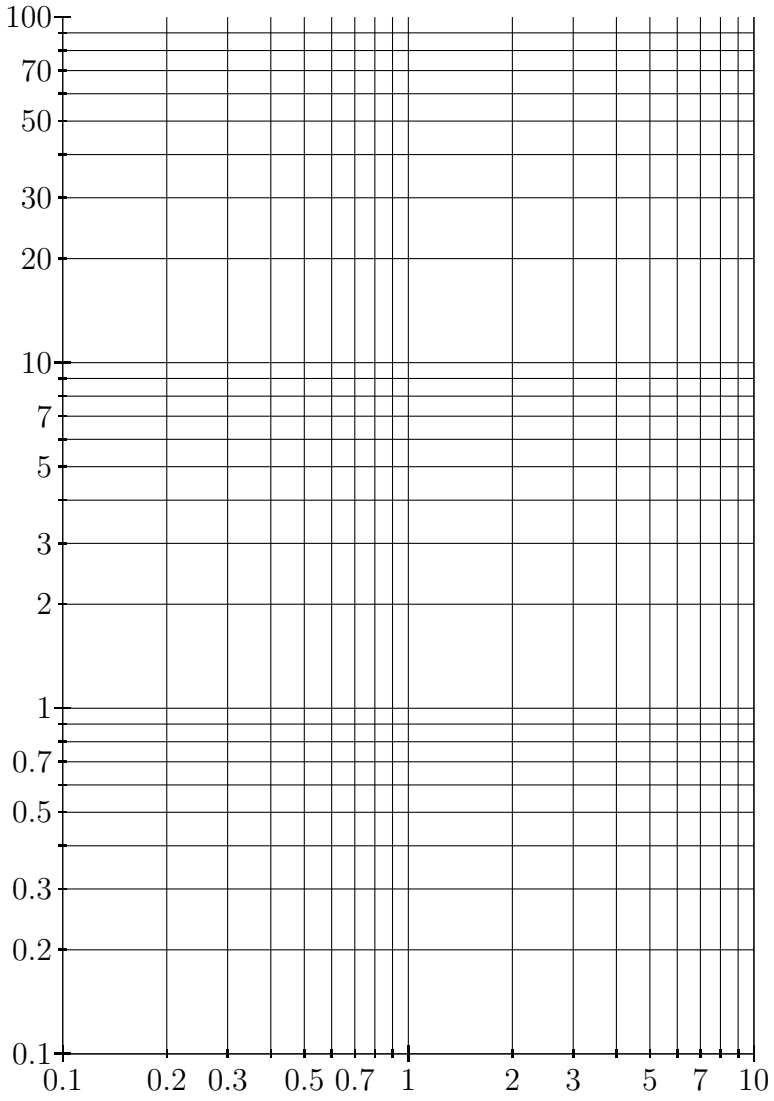
**6.** Show that $\log_{10} y = \dfrac{\ln y}{\ln 10}$. Hint: If $x = \log_{10} y$, then $y = 10^x$. Take ln of both sides.

**7.** Suppose $f(x) = 5x^{1.2}$.

(a) Fill in the table entries, and plot the resulting points in the grid:

| $x$ | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| $f(x)$ | | | | | |




(b) If $y = 5x^{1.2}$, take $\log_{10}$ of both sides. Explain the shape of your graph in (a).