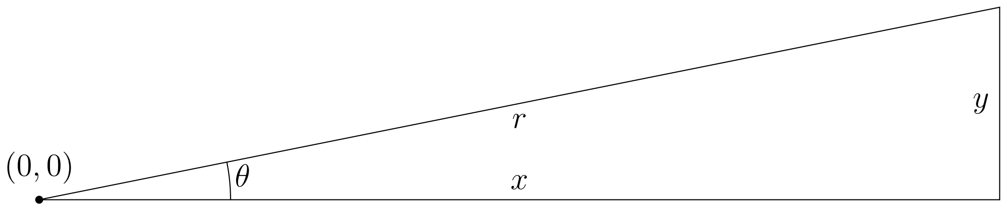
College of the Holy Cross
Math 135 (Calculus I)
Worksheet 7: Trigonometry, Polar Coordinates

1. Each part refers to the right triangle:



(a) Use similar triangles to show $\cos\theta = \dfrac{x}{r} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ and $\sin\theta = \dfrac{y}{r} = \dfrac{\text{opposite}}{\text{hypotenuse}}$.

(b) The *tangent* and *cotangent* of θ are defined to be

$$\tan\theta = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}}, \qquad \cot\theta = \frac{x}{y} = \frac{\text{adjacent}}{\text{opposite}}.$$

Show that $m = \tan\theta$ is the slope of the hypotenuse.

(c) The *secant* and *cosecant* of θ are defined to be

$$\sec\theta = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{adjacent}}, \qquad \csc\theta = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{opposite}}.$$

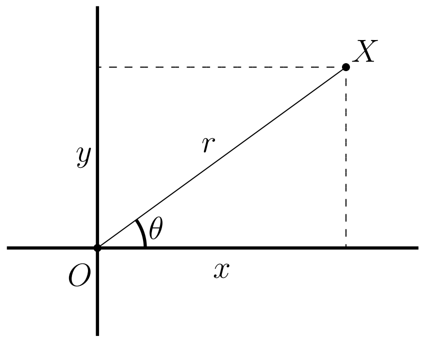
Show that $\sec\theta = \dfrac{1}{\cos\theta}$ and $\csc\theta = \dfrac{1}{\sin\theta}$.

2. Fill in the missing entries, omitting any undefined values:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$									
$\cot\theta$									
$\sec\theta$									
$\csc\theta$									

Let $O = (0,0)$ be the origin of the Cartesian plane, and let $X = (x,y)$ be a point other than the origin. Define $r = d(O,X)$ to be the *distance* from O to X, and define θ to be any angle (in radians) from the positive x-axis to the ray from O through X. The ordered pair (r,θ) is a set of *polar coordinates* for X.

3. In the diagram, the Cartesian coordinates (x,y) and the polar coordinates (r,θ) of a point X are shown. Use trigonometric functions to find formulas for x and y in terms of r and θ.



(b) Find the Cartesian coordinates of the points with polar coordinates

$$(2,0), \qquad (2,\tfrac{\pi}{6}), \qquad (2,\tfrac{\pi}{4}), \qquad (2,\tfrac{\pi}{3}), \qquad (2,\tfrac{\pi}{2}), \qquad (2,\tfrac{3\pi}{4}), \qquad (2,\pi),$$

and plot each point on a piece of polar graph paper.

4. (a) Find a set of polar coordinates for the points with Cartesian coordinates

$$(\sqrt{3},1), \qquad (2\sqrt{3},-2), \qquad (2\sqrt{2},2\sqrt{2}), \qquad (-\sqrt{3},1), \qquad (0,-5).$$

Hint: A sketch will help you find a polar angle for each.

(b) If $X = (x,y)$, find a formula for r in terms of x and y.

(c) The formulas $x = r\cos\theta$, $y = r\sin\theta$ make sense when $r \le 0$. Suppose X has polar coordinates (r,θ). Determine which of the following are also polar coordinates for X:

$$(r,\theta+2\pi), \qquad (r,\theta-2\pi), \qquad (r,-\theta), \qquad (-r,-\theta), \qquad (-r,\theta+\pi).$$