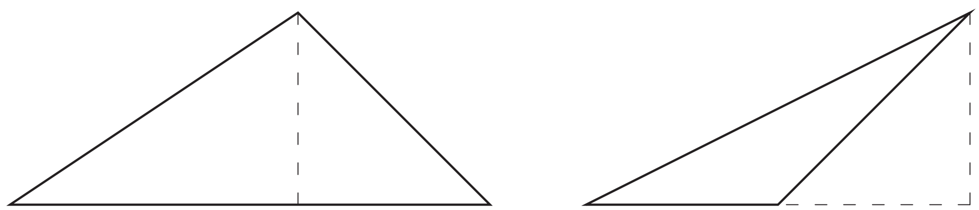
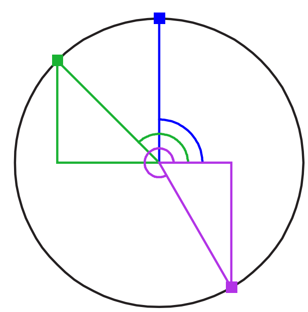
**College of the Holy Cross**
**Department of Mathematics and Computer Science**
**A Quick Review of Trigonometry**

The word *trigonometry* comes from the Greek "triangle measurement". In studying the shape of triangles, it is enough to consider *right* triangles, which contain a right angle: Just drop a perpendicular from any angle to the opposite side.



The Ancient Greeks recognized that the "shape" of a right triangle is determined by one of its acute angles, often denoted $\theta$, and that $\theta$ determines the ratios

$$\frac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta, \qquad \frac{\text{opposite}}{\text{hypotenuse}} = \sin\theta, \qquad \frac{\text{opposite}}{\text{adjacent}} = \tan\theta.$$

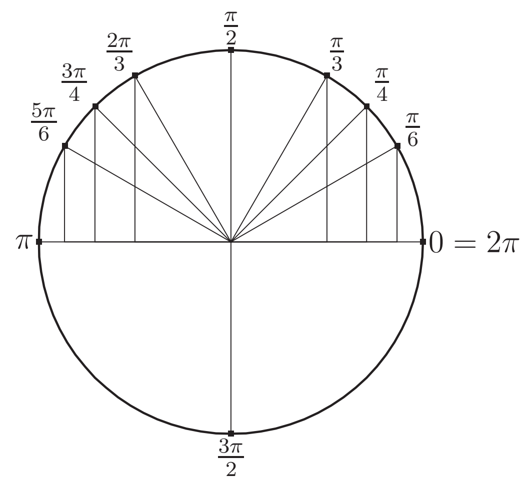


The modern view is to treat these ratios as functions of a numerical variable $\theta$ whether or not $\theta$ represents an acute angle. Since we are taking ratios of side lengths, we may assume the hypotenuse has length 1, and view our "triangles" as inscribed in the unit circle, see figure at right.
The functions cos and sin are interpreted as the horizontal and vertical coordinates of a point on the unit circle, while tan is the slope of the segment joining that point to the origin.

This discussion presupposes a numerical measure of "angle". For historical reasons originating in astronomy (a year has roughly 365 days), a full circle was divided into 360 *degrees*. In calculus, formulas work out much more nicely in *radians*, the measure of angle defined by arc length along the unit circle. Since the circle has circumference $2\pi$, there are $2\pi$ radians in 360 degrees:

$$1 \text{ radian} = \frac{360^\circ}{2\pi} \simeq 57.2957795 \text{ degrees}, \qquad 1 \text{ degree} = \frac{\pi}{180} \text{ radians}.$$

There is no geometric difference between angles that differ by an integer multiple of $2\pi$, see graphs below; the functions cos and sin are *periodic* with period $2\pi$.




A basic trigonometric identity arises from the Pythagorean theorem:

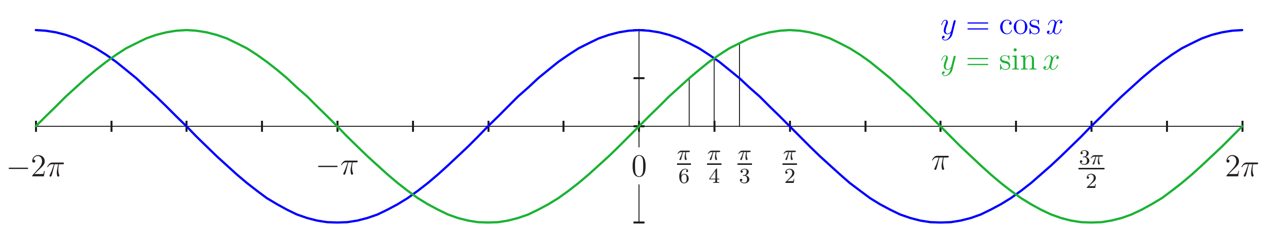
$$\cos^2\theta + \sin^2\theta = 1 \quad \text{for all } \theta.$$

Since $(\cos\theta, \sin\theta)$ is a point on the unit circle, the preceding equation expresses the familiar fact that the sum of the squares of the legs is the square of the hypotenuse.

Common angles you should know (with their sines and cosines) include

| $\theta$ | $30° = \frac{\pi}{6}$ | $45° = \frac{\pi}{4}$ | $60° = \frac{\pi}{3}$ | $90° = \frac{\pi}{2}$ | $120° = \frac{2\pi}{3}$ | $135° = \frac{3\pi}{4}$ | $150° = \frac{5\pi}{6}$ | $180° = \pi$ |
|---|---|---|---|---|---|---|---|---|
| $\cos\theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | $0$ | $-\frac{1}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-1$ |
| $\sin\theta$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | $0$ |

The values of the trig functions for these angles can be deduced from elementary geometry. For example, a right triangle with acute angle $\frac{\pi}{4} = 45°$ is isoceles, so its legs have the same length $x$. The Pythagorean theorem says that $x^2 + x^2 = 1$, or $x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.




Two additional trig identities are useful for integration:

$$\cos^2\theta = \tfrac{1}{2}(1 + \cos 2\theta), \qquad \sin^2\theta = \tfrac{1}{2}(1 - \cos 2\theta).$$

You may deduce these identities from the period ($\pi$), amplitude ($\frac{1}{2}$), and average height ($\frac{1}{2}$, the dashed red line) of $\cos^2$ and $\sin^2$: