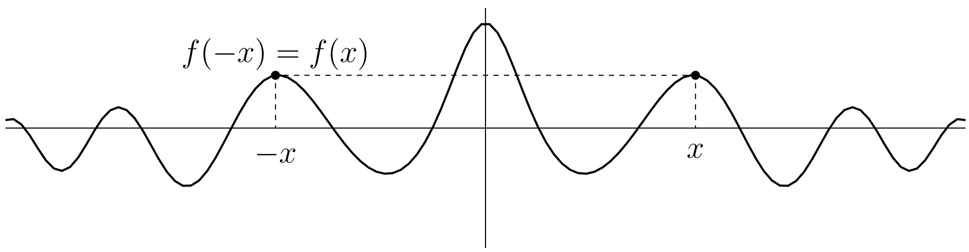
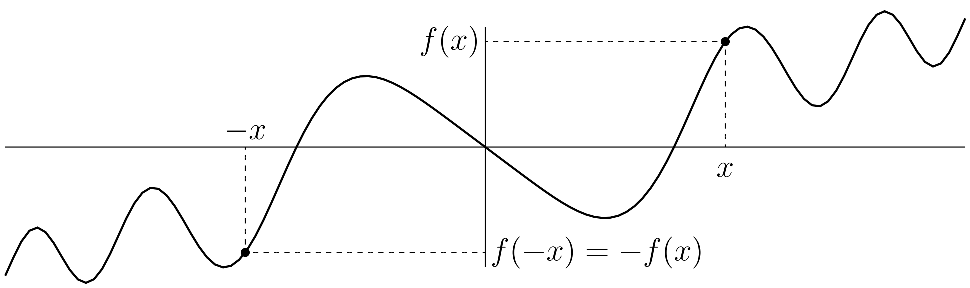
College of the Holy Cross
Even and Odd Functions

A function f is said to be *even* if $f(-x) = f(x)$ for all x in the domain of f. Geometrically, the graph of an even function remains the same under a reflection about the vertical axis:



A function f is said to be *odd* if $f(-x) = -f(x)$ for all x in the domain of f. Geometrically, the graph of an odd function remains the same under a half-turn rotation about the origin:



Most functions are neither even nor odd. However, every function f whose domain is symmetric about 0 can be written uniquely (in exactly one way) as the sum of an even function and an odd function. For example, if $f(x) = e^x$, then the functions

$$E(x) = \tfrac{1}{2}(e^x + e^{-x}), \qquad O(x) = \tfrac{1}{2}(e^x - e^{-x}),$$

are even and odd, respectively (why?), and their sum is e^x. Similar formulas decompose an arbitrary function into even and odd parts.

The even and odd parts of e^x are important enough to get special names and special notation: We call

$$\cosh(x) = \tfrac{1}{2}(e^x + e^{-x}) \quad \text{and} \quad \sinh(x) = \tfrac{1}{2}(e^x - e^{-x})$$

the *hyperbolic cosine function* and the *hyperbolic sine function.*

These functions satisfy properties closely related to the circular trig functions. You may enjoy showing that $\cosh^2 x - \sinh^2 x = 1$, $\cosh' x = \sinh x$, and $\sinh' x = \cosh x$.