College of the Holy Cross
Math 135 (Calculus I)
Group Work 3: Derivatives and Patching
Due Monday, November 12

Do all your calculations on scratch paper. Then write up your solutions neatly, using algebraic calculations and complete sentences as appropriate, on a separate sheet of paper. **Readability will earn a 10-point bonus.**

Background A differentiable function is continuous. Further, if f is continuous at c, and if $\lim_{x\to c} f'(x) = \ell$ exists, then f is differentiable at c and $f'(c) = \ell$.

1. In each part, $f(x) = x^2 e^{-x}$.

(a) Calculate $f'(x)$, and find the interval(s) where f is increasing or decreasing. Explain why f has a maximum value for $x \geq 0$.

(b) Calculate $f''(x)$, and find the interval(s) where f is convex or concave.

(c) Use the information you've found to sketch the graph $y = f(x)$ for $x \geq 0$. Be sure to include the coordinates of any interesting points.

2. Show that the function

$$f(x) = \begin{cases} -1 & x < -1, \\ \frac{3}{2}x - \frac{1}{2}x^3 & -1 \leq x \leq 1, \\ 1 & x < 1, \end{cases}$$

is differentiable and non-decreasing on $(-\infty, \infty)$.

3. Suppose a and b are constants, and that

$$f(x) = \begin{cases} ax + b & x < 1, \\ x + x^2 & 1 \leq x. \end{cases}$$

(a) Find all values of a and b so that f is continuous on $(-\infty, \infty)$. Hint: Focus on the one-sided limits of f at 1.

(b) Find all values of a and b so that f is differentiable on $(-\infty, \infty)$. Hint: Focus in addition on the one-sided limits of f' at 1.

(c) On a single piece of graph paper, sketch the graphs $y = f(x)$ if f is continuous and $a = -1$, $a = 0$, or $a = 1$.

4. Suppose a and b are constants, and that

$$f(x) = \begin{cases} 2x - x^2 & x < 2, \\ a + (b/x) & 2 \leq x. \end{cases}$$

(a) Find all values of a and b so that f is continuous on $(-\infty, \infty)$.

(b) Find all values of a and b so that f is differentiable on $(-\infty, \infty)$.