of coordinates, we see that the equation $f(x) = 0$ becomes a reduced cubic equation $X^3 + pX + q = 0$ (§ 42).

6. Find the inflexion tangent to $y = x^3 + 6x^2 - 3x + 1$ and transform $x^3 + 6x^2 - 3x + 1 = 0$ into a reduced cubic equation.

60. Real Roots of a Real Cubic Equation. It suffices to consider

$$f(x) = x^3 - 3lx + q \qquad (l \neq 0),$$

in view of Ex. 5 above. Then $f' = 3(x^2 - l)$, $f'' = 6x$. If $l < 0$, there is no bend point and the cubic equation $f(x) = 0$ has a single real root.

If $l > 0$, there are two bend points

$$(\sqrt{l}, q - 2l\sqrt{l}), \qquad (-\sqrt{l}, q + 2l\sqrt{l}),$$

which are shown by crosses in Figs. 18–20 for the graph of $y = f(x)$ in the three possible cases specified by the inequalities shown below the figures. For a large positive x, the term x^3 in $f(x)$ predominates, so that the graph contains a point high up in the first quadrant, thence extends downward to the right-hand bend point, then ascends to the left-hand bend point, and finally descends. As a check, the graph contains a point far down in the third quadrant, since for x negative, but sufficiently large numerically, the term x^3 predominates and the sign of y is negative.

If the equality sign holds in Fig. 18 or Fig. 19, a necessary and sufficient condition for which is $q^2 = 4l^3$, one of the bend points is on the x-axis, and the cubic equation has a double root. The inequalities in Fig. 20 hold if and only if $q^2 < 4l^3$, which implies that $l > 0$. Hence $x^3 - 3lx + q = 0$ *has three distinct real roots if and only if* $q^2 < 4l^3$, *a single real root if and only if* $q^2 > 4l^3$, *a double root (necessarily real) if and only if* $q^2 = 4l^3$ *and* $l \neq 0$, *and a triple root if* $q^2 = 4l^3 = 0$.