to be *not absolutely convergent* or *conditionally convergent.* To this latter class belong some convergent alternating series. For example, the series

$$1 - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \frac{1}{5^5} - \cdots$$

is *absolutely convergent*, since the series (C), p. 217, namely,

$$1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \frac{1}{5^5} + \cdots$$

is convergent. The series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

is *conditionally* convergent, since the harmonic series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

is divergent.

A series with terms of different signs is convergent if the series deduced from it by making all the signs positive is convergent.

The proof of this theorem is omitted.

Assuming that the ratio test on p. 219 holds without placing any restriction on the signs of the terms of a series, we may summarize our results in the following

General directions for testing the series

$$u_1 + u_2 + u_3 + u_4 + \cdots + u_n + u_{n+1} + \cdots .$$

When it is an alternating series whose terms never increase in numerical value, and if

$$\lim_{n=\infty} u_n = 0,$$

then the series is convergent.

In any series in which the above conditions are not satisfied, we determine the form of u_n *and* u_{n+1} *and calculate the limit*

$$\lim_{n=\infty} \left(\frac{u_{n+1}}{u_n} \right) = \rho.$$

I. *When* $|\rho| < 1$, *the series is absolutely convergent.*

II. *When* $|\rho| > 1$, *the series is divergent.*

III. *When* $|\rho| = 1$, *there is no test, and we should compare the series with some series which we know to be convergent, as*

$$a + ar + ar^2 + ar^3 + \cdots ; \ r < 1, \qquad \text{(geometric series)}$$

$$1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots ; \ p > 1, \qquad \text{(p series)}$$