If a variable v ultimately becomes and remains in numerical value greater than any assigned positive number however large, we say v, *in numerical value, increases without limit*, or v *becomes infinitely great*,[1] and write

$$\lim v = \infty, \quad \text{or}, \quad v \doteq \infty.$$

Infinity (∞) is not a number; it simply serves to characterize a particular mode of variation of a variable by virtue of which it increases or decreases without limit.

17. Limiting value of a function. Given a function $f(x)$.

If the independent variable x takes on any series of values such that

$$\lim x = a,$$

and at the same time the dependent variable $f(x)$ takes on a series of corresponding values such that

$$\lim f(x) = A,$$

then as a single statement this is written

$$\lim_{x=a} f(x) = A,$$

and is read *the limit of $f(x)$, as x approaches the limit a in any manner, is A.*

18. Continuous and discontinuous functions. A function $f(x)$ is said to be *continuous for $x = a$* if the limiting value of the function when x approaches the limit a in any manner is the value assigned to the function for $x = a$. In symbols, if

$$\lim_{x=a} f(x) = f(a),$$

then $f(x)$ is *continuous* for $x = a$.

The function is said to be *discontinuous* for $x = a$ if this condition is not satisfied. For example, if

$$\lim_{x=a} f(x) = \infty,$$

the function is discontinuous for $x = a$.

The attention of the student is now called to the following cases which occur frequently.

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[1] On account of the notation used and for the sake of uniformity, the expression $v \doteq +\infty$ is sometimes read *v approaches the limit plus infinity*. Similarly, $v \doteq -\infty$ is read *v approaches the limit minus infinity*, and $v \doteq \infty$ is read *v, in numerical value, approaches the limit infinity.*

While the above notation is convenient to use in this connection, the student must not forget that infinity is not a limit in the sense in which we defined a limit on p. 11, for infinity is not a number at all.