**College of the Holy Cross**, Spring Semester, 2011
**Department of Mathematics and Computer Science**
**Problem of the Week #3: Solved by Craig Richardson**

**Problem 3** A *Shinkansen* (bullet train) is about 1000 feet long (five cars of 200 feet each). When traveling the same direction, a *Shinkansen* passes a *Hankyu* train in about 15 seconds. Traveling in the opposite direction, a *Shinkansen* passes a *Hankyu* train in about 4 seconds. Assuming all trains travel at constant speed between Osaka and Kyoto, how fast does a *Shinkansen* travel, and how fast is a *Hankyu* train?

**Solution** Let $v_1$ be the speed of a *Shinkansen* and $v_2$ the speed of a *Hankyu* train, both measured in feet per second. The relative speeds are $v_1 - v_2$ (traveling the same direction) and $v_1 + v_2$ (traveling in opposite directions), so

$$15 = \frac{1000}{v_1 - v_2} \quad \text{and} \quad 4 = \frac{1000}{v_1 + v_2}.$$

Rearranging,

$$v_1 - v_2 = \frac{1000}{15} = 66.67 \text{ feet per second},$$
$$v_1 + v_2 = \frac{1000}{4} = 250 \text{ feet per second}.$$

The individual velocities are found by adding and subtracting:

$$v_1 = \tfrac{1}{2}(\tfrac{1000}{4} + \tfrac{1000}{15}) = 158.33 \text{ feet per second},$$
$$v_2 = \tfrac{1}{2}(\tfrac{1000}{4} - \tfrac{1000}{15}) = 91.67 \text{ feet per second},$$

or $v_1 \approx 108$ miles per hour, $v_2 \approx 62.5$ miles per hour.

**Notes** The top speed of the Shinkansen is probably closer to 120–130 miles per hour, but the Osaka-Kyoto segment is on the order of 20 miles long, so it's possible the trains do not reach top speed between stations.

Based on estimates made in Yokohama in March 1995, the *Shinkansen* trains slow to about 90 mph when passing through a station, but even behind the safety barriers (which are set back about six feet from the edge of the platform) they create a substantial pressure wave.