Building a catenoid model

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A catenoid is a surface of revolution obtained by revolving a catenary—the shape assumed by a hanging chain—about a horizontal axis. Remarkably, a catenoid is geometrically equivalent ("locally isometric") to a helicoid, a ruled surface shaped like a spiral staircase. This file contains templates for building a catenoid model out of paper. Each half of the catenoid model is made of six conical frustra, joined by a cylindrical band. On each succeeding page are paper templates for making one-half of a catenoid. In the top figure are a rectangular band and three frustra; in the lower figure are three frustra, two of them concentric. The larger the angle subtended by a flattened frustrum, the farther from the middle band the frustrum is attached. In the top figure, the frustra are (abstractly) numbered 3, 5, and 0, while in the bottom figure, the concentric frustra are 2 and 4, and the non-concentric frustrum is 1.

Carefully cut out all 12 frustra and the two rectangles. The dashed arcs are glue tabs; cut radial slits through each glue tab, about every 1/2 inch (10 mm), so that the dashed portions are coarsely fringed. A catenoid half is constructed by gluing the tab of frustrum $\#i$ to the outer edge of frustrum $\#(i-1)$, $i = 1, \dots, 5$; the two halves are then joined by a rectangular band.

Since the frustra meet along arcs of differing radii, the glue joint must be made gradually and with care; this is where the catenoid's curvature is concentrated. Align corresponding ends of the frustra to be joined and glue a few fringes at a time, gently bending the pieces so that their boundaries just overlap. It is probably easiest to glue each set of 6 frustra into a half catenoid, then to join the halves with a rectangular band. Using the "extra" band on the inside will provide additional support to this joint, where maximum flexing occurs when the model is bent into a helicoid.