

**Improving Teacher Quality Workshop  
College of the Holy Cross  
July 9 –26, 2012**

**FOUNDATIONAL MATHEMATICS CONCEPTS FOR THE HIGH SCHOOL TO  
COLLEGE TRANSITION**

**Funded by the Massachusetts Department of Higher Education**

**Led by Professors Cristina Ballantine and Sharon Frechette**

**Assisted by Zach Kenny (HC 2013)**

**Guest presentations by the following Holy Cross faculty:**

**Professor John Anderson, Dean of the Class of 2015  
Professor Diane Bukatko, Departments of Psychology and Education  
Professor Amber Hupp, Department of Chemistry  
Professor Matthew Koss, Department of Physics  
Professor Sara Mitchell, Department of Biology**

**and by the following teachers:**

**Susan Russo, Holy Name Central Catholic Jr. Sr. High School  
Colleen Stevens, Wachusett Regional High School**

**DAILY SCHEDULE**

Here is the format for a typical day.

8:15 – 8:30	Gather for coffee (Swords Faculty Lounge)
8:30 – 10:00	Presentation of content for the day's topic, including short hands-on activities.
10:00 – 10:15	Break.
10:15 – 12:00	Guest presentation and/or extended group activity on the day's topic; reports from group; and discussion of their approaches and idea.
12:00 – 12:45	Lunch.
12:45 – 1:30	Technology and/or pedagogy discussions.
1:30 – 2:30	Free time to work on lesson plans.

## **SCHEDULE OF TOPICS (subject to updates)**

### **WEEK 1**

#### **Monday, July 9** (Sharon, Diane)

Welcome, and pre-workshop formalities

Linear functions: Systems of Linear Equations, and The Method of Linear Programming

A common problem in business is how to allocate limited resources in the production of multiple products so as to maximize profit. Linear programming is a mathematical technique that is often used to solve these problems. Properties of linear functions are used to translate allocation decisions into questions about the geometry of sets in the plane or in higher dimensions, depending on the number of resources and products.

#### **Tuesday, July 10** (Cristina, John)

Power functions, polynomials, and rational functions: Areas and Volumes.

Area and volume formulas for regular shapes in the plane and space are among the first examples of functions after linear functions. For less regular shapes with symmetry, these formulas naturally lead to polynomials. In some physical problems, for example, in measuring and designing the acoustical properties of a room, we are led to consider rational functions.

Afternoon Discussion: Reflections on college readiness, from one of our class deans.

John Anderson is a professor of mathematics and also the Dean of the Class of 2015. He will lead a discussion on general college readiness, sharing his point of view as the dean of a class that has just completed freshman year.

#### **Wednesday, July 11** (Cristina, Colleen)

Fractional powers and logarithmic relationships: Scaling and Allometry.

Biophysical properties of organisms, such as metabolic, breathing, and heart rates, are related to the size of the organism by fractional power laws. These are conveniently expressed in terms of logarithms where they are seen as applications of best fit lines. The relationships scale with an organisms size as it grows and scale between adult organisms of different species. These ideas are the subject of allometry.

Afternoon Activity: Linear Scatter Plots.

#### **Thursday, July 12** (Sharon, Matt, Diane)

Periodic functions: Sines, Cosines, and Transformation of Functions.

Many natural phenomena, such as tides, length of daylight, and circadian rhythms, can be modeled by periodic functions. These models often require horizontal or vertical scaling, translation, or reflection. These transformations apply to functions in general, and each has both a geometric and algebraic interpretation.

Afternoon Discussion: The science of learning.

Diane will lead a discussion on research findings about how students learn, and classroom instruction methods that we can use to enhance learning and retention.

## **WEEK 2**

### **Monday, July 16** (Christina, Sara, Sue)

Exponentials and Logarithms: Growth and End Behavior of Functions.

Understanding the comparative growth of different functions is an important tool in many areas. Transformations of functions, and comparison of function characteristics are useful tools in determining which family of functions best fits a given situation. For example, the end behavior or long term behavior of functions plays an important role in consumer finance, in the growth of populations, and in the sustainable use of resources.

Afternoon Activity: Geogebra!

Sue will lead a hands-on, interactive session on how to use the free software package Geogebra. The focus will be on the geometry of symmetries in the plane.

### **Tuesday, July 17** (Sharon, Amber, Cristina)

Exponentials and logarithms (continued).

Afternoon Activity/Discussion: Mathematical modeling

Cristina will lead a hands-on, interactive session on how to identify a mathematical problem, and formulate a mathematical strategy, given a verbal description of a given situation.

### **Wednesday, July 18** (Sharon, Colleen)

Rigid motions in the plane

Translations, rotations, reflections through an axis, and glide reflections move an object about the plane without affecting its shape and size. These rigid motions may be combined to use a shape to produce a tessellation of the plane. How many possible types are there, from the viewpoint of symmetry? What happens if we also allow scaling? These questions will be studied using the tessellations of M.C. Escher.

Afternoon Activity: Assistments, and classroom clickers

Colleen will demonstrate and discuss the features of Assistments, and the usefulness of clickers in the classroom.

### **Thursday, July 19** (Cristina, Sue)

Vectors and 2x2 matrices

Functions of two variables can be viewed as functions on vectors in the plane. It can be useful to express these functions in vector/matrix notation, and to use matrix calculations to evaluate the functions. The study of linear functions of several variables is the cornerstone of linear algebra, which has many applications in economics and the sciences. Some linear and affine functions of two variables and their relationship to motions in the plane will be explored.

Afternoon Activity: Geogebra!

Sue will lead a second hands-on, interactive session on how to use the free software package Geogebra. The focus will be on modeling using data and best-fit functions.

## **WEEK 3**

### **Monday, July 23** (Cristina, Sharon)

Discrete functions: Combinatorics and Paths on Graphs.

A different sort of graph arises in the branch of mathematics known as combinatorics. These are geometric objects made of vertices and edges and interesting objects of study are paths in a graph. Many problems, including the famous traveling salesman problem, are naturally framed in terms of paths on graphs and functions whose inputs are paths.

Afternoon Discussion: Technology options.

Some questions to think about in advance: What are the ways you incorporate technology into your classes? What resources do you have available, and how does this impact your choices? What are the ways that you find technology enhances instruction and student learning?

### **Tuesday, July 24** (Sharon, Cristina)

Discrete functions: Cryptography

Mathematical ciphers convert plain text to encrypted text, or vice versa, using various types of functions. Affine ciphers encrypt one character at a time, using arithmetic modulo 26. Block ciphers encrypt more than one character at a time and may be described using matrices. A simple example is the 2x2 Hill Cipher, where encrypting and decrypting involve 2x2 matrices and vectors with two components.

Afternoon Discussion: The pros and cons of graphing calculators.

### **Wednesday, July 25**

Presentation of Lesson Plans.

### **Thursday, July 26**

Presentation of Lesson Plans (continued).

Overall assessments and post-test.

Farewell luncheon, hosted by Professor Chick Weiss, Director of the Office of Grants and Corporate & Foundation Giving.