

2012 Summer Workshop, College of the Holy Cross
Foundational Mathematics Concepts for the High School to College Transition

Day 9 – July 23, 2012

An IMP-like Introduction to Graphs

High school juniors and seniors in Massachusetts (and their families) face the problem of planning visits to colleges and universities over their week-long breaks. Many are interested in the University of Massachusetts schools (Amherst, Boston, Dartmouth, and Lowell) and the Massachusetts State Universities (Bridgewater, Fitchburg, Framingham, Salem, Westfield, Worcester) and the Maritime Academy (Buzzards Bay).

A group of Worcester high school students are given the problem of planning a visit to all these schools in a week. Here are some of the steps they took. They need some help, which is where you come in.

Step 1:

They started by going to Google (of course!) to find the distances and driving times between the various schools. This is what they found out:

School to School	Distance	Time
Boston (O) – Framingham (C)	25.2 mi	36 m
Worcester (L) – Westfield (I)	63 mi	1 h 13 m
Westfield (I) – Amherst (J)	31.4 mi	48 m
Amherst (J) – Fitchburg (A)	51.2 mi	1 h 14 m
Fitchburg (A) – Lowell (F)	33.3 mi	47 m
Fitchburg (A) – Worcester (L)	28.2 mi	37 m
Lowell (F) – Framingham (C)	40 mi	52 m
Worcester (L) – Dartmouth (R)	70 mi	1 h 17 m
Dartmouth (R) – Bridgewater (S)	33.5 mi	40 m
Bridgewater (S) – Framingham (C)	51.5 mi	59 m
Framingham (C) – Worcester (L)	21.5 mi	37 m
Boston (O) – Mass Maritime (N)	56.2 mi	1 h 2 m
Mass Maritime (N) – Dartmouth (R)	28.7 mi	36 m
Boston (O) – Bridgewater (S)	30.2 mi	41 m
Bridgewater (S) – Mass Maritime (N)	33.6 mi	42 m
Boston (O) – Salem (E)	18.7 mi	34 m
Salem (E) – Lowell (F)	33.8 mi	50 m

Job 1: Make sure you understand what this table is telling you. Can you answer these questions?

- How would you figure out how far it is from Amherst to the Mass Maritime Academy?
- Will everyone answer the first question in the same way? Why (or why not)?

Step 2:

The students decided that it was difficult to work with the data in this form and that it would be better to place it on the map. The map was cluttered so they decided to create a “stick figure” map, with dots (•) representing each school and sticks (—) representing the route from one school to another. And they thought it would be a good idea to label the dots with names and the sticks with distances and times.

Job 2: Your second job is to produce this sketch.

Mathematical terminology:

- The dots are called *vertices*.
- The sticks are called *edges*.
- A sketch of this type is called a *graph*. Notice this use of the term is different from our usual use of the term in Algebra.

Step 3:

Once the students had completed their graph (let’s try to use the mathematical terminology), they wanted to put it to use to solve their problem. They agreed they should visit every school, but they didn’t agree on much after that. Well, that’s not quite true. There were seven students in the group and they had three ideas between them. Here’s what they thought:

- Xuan Ha and Hieu thought they shouldn’t have to visit any school or pass through its city twice. They wanted to start in Worcester and end in Worcester. So Xuan Ha and Hieu said the route on the graph should pass through every vertex once, except for Worcester.
- This made Xavier and Sofjola think of a different approach. They decided they should drive over each road once, figuring they would have to get to each school along the way. So they said the route on the graph should pass along each edge once.
- Yaw and Tashara thought they should spend the least amount of time driving starting and ending in Worcester. So they said that when they added up the times for the edges that made up their trip, it should be the smallest total that one could find.
- Jennifer couldn’t decide, but she knew her Aunt, who lived in Boston, would be taking her around. So they would be starting in Boston and Jennifer wanted to know if that would make a difference.

As experienced IMP students, they know that you may have to experiment to find an answer, that the answer might be hard to find, or worse they might not be able to find it, but that they should be able to explain their thinking.

Job 3: Your third job is to help the different pairs and Jennifer answer their questions.

- For Xuan Ha and Hieu: Can you figure out a way to do what they want?
 - If so, your answer should be a list of the schools they should visit in order they should visit them.
 - If so, do Xuan Ha and Hieu have to do it the same way, or can they do it a different way?
 - If it isn't possible, can you explain why its not?
- For Xavier and Sofjola: Can you figure out a way to do what they want?
 - If so, your answer should be a list of the schools they should visit in order they should visit them.
 - If so, do Xavier and Sofjola have to do it the same way, or can they do it a different way?
 - If it isn't possible, can you explain why its not?
- For Yaw and Tashara: Can you figure out a way to do what they want?
 - If so, your answer should be a list of the schools they should visit in order they should visit them and the total time spent driving.
 - If so, do Yaw and Tashara have to do it the same way, or can they do it a different way?
 - If it isn't possible, can you explain why its not?
- For Jennifer:
 - Does she have to redo everyone else's work, or will she be able to use their solutions?
 - If so, explain why and provide her with a solution.
 - If not, explain why not.