2012 Summer Workshop, College of the Holy Cross Foundational Mathematics Concepts for the High School to College Transition

Warm-up Exercise

Assume all the usual rules of algebra (and no new rules) apply in the following. Define three functions as follows:

$$\&(\heartsuit) = \%^{\heartsuit}, \qquad \#(\spadesuit) = \spadesuit \cdot \spadesuit, \qquad \$(\clubsuit) = \clubsuit + \clubsuit.$$

Which of the following are valid statements? If a statement is not valid, what is the correct right side?

1.
$$\&(\heartsuit + \heartsuit) = \%^{\$(\heartsuit)}$$

2.
$$(\# \circ \$)(\heartsuit) = (\$ \circ \#)(\heartsuit)$$

3.
$$(\& \circ \$)(\eta) = (\# \circ \&)(\eta)$$

4.
$$(\$ \circ \&)(\aleph) = (\& \circ \$)(\aleph)$$

5.
$$\#(\$^{-1}(\spadesuit)) = \frac{\#(\spadesuit)}{\$(2)}$$

6.
$$(\$ \circ \&^{-1})(\clubsuit) = (\&^{-1} \circ \#)(\clubsuit)$$