2012 Summer Workshop, College of the Holy Cross
Foundational Mathematics Concepts for the High School to College Transition

Day 5 – July 16, 2012

End Behavior of Functions, Rates of Growth, Exponential Functions

(From Stewart's Calculus book) Suppose you are offered a job that lasts one month. How would you like to be paid?

(i) One million dollars at the end of the month.

(ii) One cent on the first day, two cents on the second day, four cents on the third day, eight cents on the fourth day, etc. (after the first day, each day you are paid the double amount you were paid the previous day).

If a month has 30 days, under option (ii), on the last day alone you will be paid 2^{29} cents, that is 536.870.912 cents or \$5.368.709 and 12 cents.

It is much better to take the second option.

World population (Source: Wikipedia)

Population (in billions)	1	2	3	4	5	6	7	8	9
Year	1804	1927	1960	1974	1987	1999	2012	2027	2046

It looks like in recent times the population doubled approximately every 40 years.

Let $t = 0$ represent the year 1960 and $P(t)$ represent the population of the Earth (in billions of people) t years after 1960. Then,

$P(0) = 3$

$P(40) = 3 \cdot 2$

$P(80) = 3 \cdot 2^2$

$P(120) = 3 \cdot 2^3$

$\vdots$

$P(40 \cdot n) = 3 \cdot 2^n$

If $t = 40n$, then $n = \dfrac{1}{40}t$. So,

$P(t) = 3 \cdot 2^{\frac{1}{40}t}$ or $P(t) = 3 \cdot \left(2^{\frac{1}{40}}\right)^t$ or $P(t) \approx 3 \cdot 1.0175^t$.

Since $s^{\frac{1}{40}} \approx e^{0.0173}$, we can also write $P(t) = 3 \cdot e^{0.0173t}$.

How does the population change from one year to the next? From 1978 to 1979? From 2012 to 2013?

$$\frac{P(19)}{P(18)} \approx \frac{3 \cdot 1.0175^{19}}{3 \cdot 1.0175^{18}} \approx 1.0175$$

$$\frac{P(53)}{P(52)} \approx \frac{3 \cdot 1.0175^{53}}{3 \cdot 1.0175^{53}} \approx 1.0175$$

So, $P(t+1) \approx 1.0175P(t) = P(t) + 0.0175P(t)$ or

$P(t+1) \approx 1.0175P(t) = P(t) + 1.75\%P(t)$.

The population increses by 1.75% of its size every year.

This model is not very realistic since natality rates are changing. The logistic model is more appropriate.